

# Polytropic models of filamentary molecular clouds: structure, stability and magnetic fields

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**Abstract.** Recent observations made by the Herschel Space Observatory and the Planck Satellite have shown that molecular clouds are made by networks of filaments, shaped by interstellar turbulence and magnetic fields. We analyse the stability of filamentary molecular clouds with the help of cylindrical polytropic models with helical magnetic fields.

## 1. Introduction

The density profiles of filaments in the radial direction are characterised by a flat-density inner part of size  $\varpi_{\text{flat}} = (0.03 \pm 0.02)$  pc and a power-law envelope extending  $\sim 10 \varpi_{\text{flat}}$  (André et al. 2013). A convenient parametrisation is a softened power-law profile

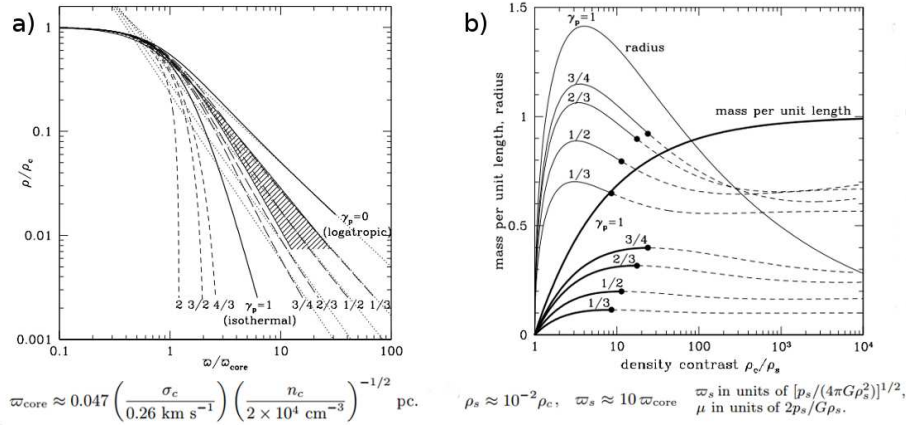
$$\rho(\varpi) = \frac{\rho_c}{[1 + (\varpi/\varpi_{\text{flat}})^2]^{\alpha/2}}, \quad (1)$$

Where  $\rho_c$  is the central density and  $\alpha$  is a parameter. If  $\alpha = 4$  this is an exact solution of the equation of hydrostatic equilibrium for self-gravitating isothermal cylinders (Ostriker 1964). However, the power-law slope measured in a sample of filaments is on average  $\alpha = 1.6 \pm 0.3$  (Arzoumanian et al. 2011), and this implies that the filaments are not in isothermal hydrostatic equilibrium. A more general class of hydrostatic models are cylinders that obey a polytropic equation of state, where the gas pressure  $p$  (due to thermal or non thermal motion) is parametrised as

$p = K\rho^{\gamma_p}$ , where  $\gamma_p > 0$  is the polytropic exponent, and the constant  $K$  is a measure of the cloud's entropy. We solved the equations for the hydrostatic equilibrium using a polytropic equation of state. Fig. 1a shows solutions of density profiles for various cylindrical polytropes of positive and negative index. In order to compare different models we normalise the radial coordinate to the same length scale.

The observed radial density profiles are well reproduced by cylindrical polytropes with  $1/3 \lesssim \gamma_p \lesssim 2/3$ , and this implies that other contributions to the support against gravity are needed. Support can come from either outward increasing temperature gradients, or from the presence of a dominant non thermal contribution to the pressure. In the former case the predicted gas temperature at the surface would be unrealistic. Non thermal support could come from the magnetic field.

The evolution of a real filament can be analysed as a series of polytropic magnetostatic solutions where the magnetic field has a generic helical geometry (Toci & Galli 2015b). We solved general equations for magnetised



**Fig. 1.** (a) Normalised radial density profiles of polytropic cylinders. *Thick solid curves* are the isothermal ( $\gamma_p = 1$ ) and the logatropic ( $\gamma_p = 0$ ) density profiles. *Dotted curves* are the singular scale-free solutions. The *hatched area* corresponds to the observed mean density profile. For the normalisation  $n_c$  is the central density of the filament and  $\sigma_c$  the velocity dispersion. (b) Mass per unit length  $\mu$  (*thick curve*) and radius  $\varpi_s$  (*thin curves*) of bounded cylindrical polytropes as a function of the centre-to-surface density contrast  $\rho_c/\rho_s$ . Dots indicate critical points. The stable and unstable parts of each curve are shown by *solid* and *dashed* curves, respectively (from Toci & Galli 2015a).

filamentary clouds. In this case, according to the power-law slope, the magnetic field affects the radial density profile in different ways. In the range  $\gamma_p \geq 0$ , the magnetic field can either support or compress the cloud according to the value of the ratio between the toroidal and the poloidal component, the pitch angle  $\delta$ . Pure poloidal fields (or with small  $\delta$ ) provide support to the cloud, allowing higher values of the envelope density compared with the thermal case.

Cylindrical polytropes are known to be unstable to longitudinal perturbation (Ostriker 1964). We studied the condition for stability to radial perturbation determined by solving the equation for radial motion for small perturbation in cylindrical symmetry. Fig. 1b shows the radius and the mass per unit length  $\mu$  of cylindrical polytropes with various values of  $\gamma_p$  as a function of the centre-to-surface density contrast  $\rho_c/\rho_s$ . Isothermal solution are always stable, while for  $0 \lesssim \gamma_p < 1$  increasing  $\rho_c/\rho_s$  the filaments first expands then contracts until a critical value. Equilibria exist above the critical value but are unstable.

## 2. Conclusions

The observed radial density profiles are well reproduced by negative-index polytropes with  $1/3 \lesssim \gamma_p \lesssim 2/3$ , indicating non-thermal pressure components, perhaps resulting from a superposition of small-amplitude Alfvén waves (corresponding to  $\gamma_p = 1/2$ ). For pressure-bounded cylindrical polytropes, the mass per unit length depends on the boundary conditions and is not limited as in the isothermal case. Magnetised cylindrical polytropes with a helical magnetic field have the Lorentz force directed outward or inward depending on whether the value of the pitch angle is above or below a critical value which depends only on the polytropic exponent.

## References

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